Why neg-raising requires stativity*

Paloma Jeretič[1] and Deniz Özyıldız[2]

[1]ZAS Berlin [2]Universität Konstanz

1 Introduction

The neg-raising (NR) inference is the inference from sentences like (1a) to ones like (1b), where negation is expressed on the matrix verb, but understood to negate the embedded proposition.

(1) a. Jo doesn't think that Su paid rent. b. Jo thinks that Su didn't pay rent.

The inference can be drawn with some predicates, like 'think,' 'believe' and 'want,' but not with others, like 'be certain,' 'claim' or 'wish.' A curious generalization, in (2), further links its availability to lexical aspect, and tells us what kinds of (uses of) predicates *cannot* trigger it (modified from Bervoets 2014: p. 112). Why this should hold is not yet well understood.[1]

(2) All NR predicates are stative, and in case an eventive counterpart of a NR predicate exists, this eventive counterpart does not trigger the NR inference.

In this paper, we focus on thought reports in French (which differentiates in the past between imperfective and perfective) and show that these are stative and NR in the imperfective, but eventive and non NR in the perfective—as expected per the generalization (Sect. 3). Adopting independently motivated assumptions about lexical and grammatical aspect (Sect. 2), we show that an analysis of the NR inference as a Scaleless Implicature (Jeretič 2021, 2022, see also Mirrazi and Zeijlstra 2021), but not as derived from an Excluded Middle presupposition (Bartsch 1973; Gajewski 2005), predicts this observation (Sect. 4). This result is a step towards understanding (2) in its full generality, as well as how logical and eventuality related properties of attitude predicates (like NR and aspect) interact.

2 The perfective and eventivity (background on aspect)

Much like their non attitude counterparts, attitude predicates can be categorized into ones that describe states ('know,' 'be certain'), and ones that describe events ('find out').[2] Often, we find that attitude predicates *alternate* between stative and eventive: In English, *think* may give rise to an ongoing state reading in the simple present, or to an ongoing event reading in the present

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[1]The first mention of this generalization that we know of is in Horn (1978: p. 206), who attributes it to Polly Jacobson. It seems to be found in early sources on NR under various guises, which oppose 'parenthetical,' 'performative' or 'metaphorical' uses to the 'literal' uses of a predicate, where only the former are NR (Horn and Bayer, 1984; Lakoff, 1969; Prince, 1976). The link with aspect receives renewed attention in work by Bervoets (2014, 2020), Xiang (2014), Özyıldız (2021) and Bondarenko (2022).

[2]More accurately, predicates taken together with their arguments and modifiers are categorized in this way.

progressive, in (3). Example (4) shows that a stative like *be mad* cannot occur in the simple present, and an eventive like *draw a circle*, in the progressive (Dowty 1979, a.o.).

(3) Calypso {thinks, is thinking} that it's raining.

(4) a. Zoe {is, #is being} mad. b. Zoe {#draws, is drawing} a circle.

These stative/eventive alternations are varied; for concreteness, we focus on the alternation between stative and eventive thought reports introduced by French *penser* (our findings extend to *croire*, 'believe'). We choose French because predicates may occur in the imperfective *imparfait* or the perfective *passé composé*, the latter forcing eventive interpretations.

(5) Hier, Calypso {pensait, a pensé} qu'il pleuvait.
Yesterday, Calypso thought$_{\{\text{IPFV, PFV}\}}$ that it was raining.

In general, both eventive and stative predicates, in (6a) and (7a), may occur in the imperfective, the former giving rise to an ongoing event, and the latter, to an ongoing state reading. In the perfective, however, while eventive predicates are acceptable, in (6b), statives are usually odd, in (7b). They become acceptable through contextual support, or with adverbs like *soudainement* ('suddenly') or *deux fois* ('two times'). When they are, the state that they describe is interpreted as bounded. We therefore rely on the perfective to identify bounded thought reports.[3]

(6) a. Hier, Zoé dessinait un cercle.
Yesterday, Zoé was drawing a circle.
b. Hier, Zoé a dessiné un cercle.
Yesterday, Zoé drew a circle.

(7) a. Hier, Zoé était en colère.
Yesterday, Zoé was$_{\text{IPFV}}$ mad.
b. Hier, Zoé a été en colère #(2 fois).
Yesterday, Zoé was$_{\text{PFV}}$ mad twice.

This restriction is at first sight unexpected. A common meaning for the perfective, attributed to Klein (1994), places the the runtime $\tau(e)$ of an event e within a contextually determined reference time R. Applied to *be mad*, we get the statement in (8), which should be satisfied by any state of Zoé being angry whose runtime is contained within yesterday. This is not odd in itself as some states, like being angry, hungry, etc., can be very temporary.

(8) $[\![\text{Zoé a été en colère}]\!] = \exists e.\tau(e) \subseteq \textit{yesterday} \wedge \textit{angry}(\textit{zoé}, e)$

We follow Bary (2009) and Homer (2021) in assuming that statives are odd in the perfective because the perfective imposes a restriction on the eventuality descriptions it combines with, namely, that these be quantized. As defined in (9), a quantized predicate is one which, when it is true of an event, is false of any of its subevents. The perfective is defined, in (10a), with quantization as a definedness condition on its predicate argument, cf. the imperfective in (10b).

(9) $\text{QUA}(P)$ iff $\forall e \forall e'[(P(e) \wedge e' \sqsubset e) \rightarrow \neg P(e')]$

(10) a. $\text{PFV} := \lambda P : \text{QUA}(P).\exists e.P(e) \wedge \tau(e) \subseteq R$ b. $\text{IPFV} := \lambda P.\exists e.P(e) \wedge R \subseteq \tau(e)$

A predicate like *draw a circle* is quantized as no completed drawing of a circle is a proper subevent of another one, and hence, acceptable in the perfective. Stative predicates, however, are not quantized, and are divisive. This is defined in (11a): an eventuality true of a stative has all its subevents also true of that stative. To combine a stative with the perfective, then, one must first derive a quantized predicate from the stative. There are different ways of doing

[3] *Penser* has the option of being eventive in the imparfait as well. We have reason to think that these cannot be NR either, much like progressive thought reports in English (Özyıldız, 2021). We also leave the question of why bare *penser* sounds better in the perfective than bare *être en colère* to another occasion.

so, falling under the general notion of aspectual coercion (Moens and Steedman, 1988). We adopt Bary's MAX operator, defined in (11b), which takes a predicate and maximizes it, such that MAX(P) is true of P-events that have no proper P-subevents. Applied to a stative like 'be angry' or 'think p', it returns a predicate true of bounded events of being angry and thinking p. As such predicates are quantized, they are now free to combine with PFV as defined in (10a).

(11) a. P is stative iff $\forall e[P(e) \rightarrow \exists e'[e' \sqsubset e \wedge \forall e''[e'' \sqsubset e \rightarrow P(e'')]]]$
b. $\textsc{max} := \lambda P \lambda e.P(e) \wedge \forall e'[e \sqsubset e' \rightarrow \neg P(e')]$ (Bary, 2009)

Implicit in this discussion is the assumption that *think* is stative at its core, and its eventive counterpart is derived from it via aspectual coercion. (There are alternatives, see Özyıldız 2021.)

Finally, it is crucial for us that negated statives involve aspectual coercion as well. One reason for thinking so is that the negation of a stative in the perfective remains odd, in (12).[4]

(12) Hier, Paul {n'était pas, #n'a pas été} en colère.
Yesterday, Paul was$_{\{\text{IPFV, PFV}\}}$n't mad.

3 NR requires stativity

Thought reports can be stative or eventive. If it is right that NR only arises with statives, we should find that stative thought reports may (and in fact tend to) give rise to the inference, but that their eventive counterparts may not. This is what we proceed to show now.[5]

The data supporting this result is delicate, and we first exercise our intuition on naturally occurring examples. Example (13a) is about carelessness in adopting children from areas struck by disaster, and (13b) reports on an unsuspecting shop owner, moments before she got robbed. Both examples feature perfective *penser* and can be translated naturally as "it did not occur to X that p," which does not involve NR. This is particularly clear in (13a), as it is surely false that the subjects have (had) the thought that adopted children would *not* grow up.

(13) a. On n'a pas pensé que les enfants adoptés deviendraient des adultes.
We didn't think$_{\text{PFV}}$ that adopted children would become adults.
b. Elle n'a pas pensé que c'était quelqu'un de mal intentionné et a ouvert.
She didn't think$_{\text{PFV}}$ that it was an ill-intentioned person, and opened the door.

We now turn to constructed examples, that show that at least in some contexts, NR is natural in the imperfective, but not in the perfective. First we use a continuation based test for NR inspired by Collins and Postal's (2014) "Do you agree?" test. The continuations in (14) attribute a positive thought to Isa's mother, which is recovered anaphorically. If the preceding utterance can be NR, it should be able to provide one and feel natural. In the absence of NR, the preceding utterance should entail the absence of a thought, and the continuation should feel unnatural, as it requires there to be one.

(14) Quand elle est entrée dans une église pour la première fois,
When she went into a church for the first time,

[4] Another one is that actuality entailments survive under negation: *Marie n'a pas pu dormir*, 'Marie was$_{\text{PFV}}$n't able to sleep,' entails Marie didn't sleep. According to Homer (2021), these arise because another ('actualistic') way of coercing statives into being quantized for the perfective (necessary in particular for certain modals). If the quantization requirement were lifted, we would not see actuality entailments under negation.

[5] To be clear, similar data is found in the references cited in the introduction. But the generalization is striking enough to warrant the discussion of additional corroborating—and indeed also limiting—examples.

a. Isa {pensait, a pensé} que Dieu n'existait pas. Sa mère pensait ça aussi.
Isa thought that God didn't exist. Her mom thought that as well.

b. Isa {ne pensait pas, #n'a pas pensé} que Dieu existait. Sa mère pensait ça aussi.
Isa didn't think that God existed. Her mom thought that as well.

c. Isa ne prétendait pas que Dieu existait. #Sa mère pensait/prétendait ça aussi.
Isa wasn't claiming that God existed. #Her mom thought/claimed the same.

In the control sentences with embedded negation, in (14a), the choice of the perfective vs. the imperfective does not affect the felicity of the continuation. But, when negation is pronounced on the matrix verb, in (14b), the continuation is acceptable with *penser* in the imperfective, but not in the perfective. This suggests that NR becomes unavailable then. We compare this state of affairs to the oddness that arises with a non NR predicate in the imperfective, in (14c).

Second, we test NR with strong NPIs. These are known to be licensed under local negation or negated NR predicates, but not under negated non-NR predicates (Horn 1978, a.o.). What we see in (15a) is that embedded negation can license the strong NPI *de la nuit* regardless of whether *penser* is in the perfective or the imperfective. With matrix negation, the NPI is licensed when the *penser* is in the imperfective, but not when it is in the perfective. Perfective *penser* patterns similarly to the non NR predicate *prétendre*, in (15c). This, again, is evidence that the perfective disrupts the availability of the NR inference.

(15) a. Ce matin, Jo {pensait, a pensé} que Al n'avait pas dormi de la nuit.
This morning, Jo thought that Al hadn't slept a wink.

b. Ce matin, Al {ne pensait pas, #n'a pas pensé} que Al avait dormi de la nuit.
This morning, Jo didn't think that Al had slept a wink.

c. Ce matin, Jo ne prétendait pas que Al avait dormi (#de la nuit).
This morning, Jo wasn't claiming that Al had slept a wink.

Despite the relative sharpness of these contrasts, we make a qualification to the generality of the claim that the perfective disrupts NR with thought reports—we believe it should be put on the record, so that we may return to it at another occasion. First, there are contexts in which negated perfective *think* is, or appears to be, NR. In (16), for instance, Jo is committed to the view that Al's work was not convincing. Note also that the continuation succeeds.

(16) Al: Alors, t'en as pensé quoi de mon travail?
So what did you think of my work?

Jo: J'avoue que j'ai pas pensé qu'il était très convaincant. Zoé a pensé ça aussi.
I admit that I didn't think that it was very convincing. Zoé thought that too.

Pending a more thorough empirical investigation, we mention a couple of ways of squaring this fact with the data in (13), (14) and (15), which suggested the absence of the inference in the perfective. First, and we thank Vincent Homer (p.c.) for pointing out this option to us, Jo's utterance could be euphemistic. Second, it could be that Al's question sets up the assumption that Jo is opinionated about whether the work was convincing (this is *Adressee Competence* associated with information seeking questions (Farkas, 2022)). And if one utters $\neg$*think* p against the opinionated background *think* $p \vee$ *think* $\neg p$, the stronger meaning *think* $\neg p$ is derived. This is Bartsch's (1973) pragmatic Excluded Middle presupposition, which we elaborate on in the next section. Another solution would be, if we are right in this paper's claim that MAX is the reason NR is disrupted, that there are other ways of making stative thought reports compatible with the perfective that do not disrupt NR.

Finally, we want to note that the generalization concerns belief predicates, but not all canonically NR predicates. In particular, at least the desire predicate *vouloir* ('want') triggers the inference as strongly in the perfective as it does in the imperfective.

(17) Ce matin, Axelle n'a pas voulu que Salomé dorme [de tout l'après-midi]$_{\text{NPI}}$.
This morning, Axelle didn't want$_{\text{PFV}}$ that Salomé sleep all afternoon.

Regarding this, there are reasons to think that NR arises from different mechanisms depending on the predicate, as discussed in Jeretič (2021), which makes it possible for the perfective to interfere with *think*'s NR inference, without interfering with *want*'s.

4 An explanation

***Think* as an eventuality predicate** To combine aspectual operators with *think*, we incorporate its Hintikkan (1969) definition into an eventuality predicate, as that is what these compose with. The function *dox* is then defined to return a set of worlds compatible with an agent's belief *states*, rather than their propositional beliefs (following Hacquard (2006), a.m.o.).

(18) a. $dox := \lambda x \lambda e.\{w \,|\, e \text{ is a belief state of } x \text{ and } w \text{ is compatible with the content of } e\}$
b. $[\![\textsf{think}]\!] := \lambda p \lambda x \lambda e.dox(x, e) \subseteq p$

Note that the output of *dox* when its eventuality argument is saturated by an event that is not a belief state e of x is the empty set. Before composing with aspect, then, "Zoé think that it's raining," is a predicate true of Zoé's belief states whose content entails that it is raining, or, those at which she believes this.

(19) $[\![\textsf{think}]\!]([\![\textsf{that it's raining}]\!])([\![\textsf{Zoé}]\!]) = \lambda e.dox(zoe, e) \subseteq rain$

How the Excluded Middle presupposition fares A prominent set of accounts of NR makes use of an Excluded Middle assumption.[6] We explore whether two of them—EM as a pragmatic (Bartsch, 1973) or a semantic (Gajewski, 2005) presupposition—can be extended to derive our facts. Bartsch seminally proposes that *think* triggers a pragmatic EM presupposition (PP). She formulates it, as is usual for attitude reports, in the simple present, as in (20).

(20) *PP:* Jiu thinks it's raining or Jiu thinks it's not raining.
PP & "Jiu doesn't think it's raining" ⇒ Jiu thinks it's not raining.

While she does not discuss cases where the thought report is in another tense or aspect, it is natural to imagine that a past perfective report, for example, also has its PP in the past perfective, in (21). Nothing then stops NR from being derived, just like it is in the present.

(21) *PP:* Jiu thought$_{\text{PFV}}$ it rained or Jiu thought$_{\text{PFV}}$ it didn't rain.
PP & "Jiu didn't think$_{\text{PFV}}$ it rained" ⇒ Jiu thought$_{\text{PFV}}$ it didn't rain.

This result is undesirable. One could say in response that the EM is not presupposed with perfective *think*. After all, there are contexts where NR is suspended. See (22), from Bartsch.

(22) Peter knows about Brutus and Caesar, but doesn't know if the two lived at the same time. He neither thinks that Brutus killed Caesar, nor that Brutus didn't kill Caesar.

[6] We do not discuss syntactic NR (Collins and Postal, 2014; Crowley, 2019), as it is unclear to us how different choices of matrix verb aspect could allow or block the syntactic raising of negation from an embedded clause.

When it is explicitly stated that the attitude holder is unopinionated, the EM cannot be presupposed, and NR is not derived. Perhaps, then, contexts in which perfective thought reports are uttered are (for some reason) typically ones in which the attitude holder is not unopinionated, hence ones in which NR is not derived. This is even consistent with our example (16), which could be argued to be NR despite the perfective because the context makes opinionatedness explicit. However, this pragmatic account is jeapordized by examples like (23) (and also (13)). Here, the speaker says that they did not (perfectively) think that the Earth was round.

(23) Ce matin, comme à peu près tous les matins, je n'ai pas pensé que la Terre était ronde.
This morning, like almost every morning, I didn't think$_{\text{PFV}}$ that the Earth was round.

This sentence is not NR, despite the fact that world knowledge dictates that the speaker must be opinionated about whether the Earth is round. This suggests that whatever is hindering NR in the perfective, it is not insufficient contextual support for the EM presupposition.

We now consider the more widely adopted view that EM is a *semantic* presupposition, as first proposed by Gajewski (2005), building on a suggestion by Heim (2000). As far as we know, there is no discussion of how the presupposition interacts with aspectual operators. We choose to recast the EM presupposition for *think* in an event-based framework as follows:

(24) $\mathsf{a\ think\ p} := \lambda e.dox(a,e) \subseteq p$ *presupposes:* $\forall e'[dox(a,e') \subseteq p \vee dox(a,e') \subseteq \neg p]$[7]

We will show that this presupposition predicts a contrast between imperfective and perfective, where NR is derived in the former but not the latter. Despite this welcome result, the result for the perfective is inaccurate, as it is different from a simple lack of NR. First, let us see how the presupposition in (24) correctly derives NR in the imperfective aspect.

(25) NEG [IPFV [$\mathsf{a\ think\ p}$
ass: $\neg\exists e[R \subseteq \tau(e) \wedge dox(a,e) \subseteq p]$ $\equiv \forall e[R \subseteq \tau(e) \to \neg(dox(a,e) \subseteq p)]$[8]
psp: $\forall e'[dox(a,e') \subseteq p \vee dox(a,e') \subseteq \neg p]$ $\therefore \forall e[R \subseteq \tau(e) \to dox(a,e) \subseteq \neg p]$

A sentence like 'a didn't think$_{\text{IPFV}}$ p' as in (25) entails that all belief states of a whose runtime contains the reference time R have as content $\neg p$. If a has any R-containing belief state (which will generally be true), then it will have as content $\neg p$. This entails a NR inference.[9]

We now will see that in the perfective, the EM presupposition predicts disruption of NR, but not as desired. The disruptor is not perfective per se, but rather the quantization requirement that is necessary for any predicate to compose with the perfective, discussed in section 2.

(26) NEG [PFV [MAX [$\mathsf{a\ think\ p}$
ass: $\neg\exists e[\tau(e) \subseteq R \wedge dox(a,e) \subseteq p \wedge \forall e'[e \sqsubset e' \to \neg(dox(a,e') \subseteq p)]]$
$\equiv \forall e[\tau(e) \subseteq R \wedge \forall e'[e \sqsubset e' \to \neg(dox(a,e') \subseteq p)] \to \neg(dox(a,e) \subseteq p)]$
psp: $\forall e'[dox(a,e') \subseteq p \vee dox(a,e') \subseteq \neg p]$
$\therefore \forall e[\tau(e) \subseteq R \wedge \forall e'[e \sqsubset e' \to dox(a,e') \subseteq \neg p] \to dox(a,e) \subseteq \neg p]$

The MAX operator introduces a conjunct that contains the term $\neg(dox(a,e') \subseteq p)$. Together with the EM presupposition, this term is equivalent to $dox(a,e') \subseteq \neg p$. The result, instead of non-NR

[7] Another option we considered was the *existence* of an event of opinionatedness, whose runtime includes reference time. The predictions are almost identical, except if the domain of the subject's belief states is empty.

[8] Note that our semantics for the perfective and the imperfective under negation makes for truth conditions that are not restrictive enough. Our results should not be affected by this issue.

[9] This result is not exactly equivalent to the meaning of a sentence with negation in the lower clause, that would state the *existence* of a belief state of a of $\neg p$. Yet if we keep the events in our domain close to R, we get near equivalence, since if there is a belief state of $\neg p$ that contains R, all belief states that contain R are $\neg p$.

belief claim, is a tautology, given that ‘think’ is a stative predicate. It reads: every e running in R such that all superstates of e are $\neg p$-beliefs is also a $\neg p$-belief. This already follows from stativity (see (11a)) of *think*: e, as a substate of a $\neg p$-belief state, is also a $\neg p$-belief state.[10]

Furthermore, we note that we also get an unexpected result in a positive quantized *think* utterance. In this case, the term $\neg(dox(a, e') \subseteq p)$ is still present, and again neg-raised. The result in (27) is a contradiction: it states that there is a p-belief state of a whose superevents are $\neg p$-belief states of a. A state that makes P true has all of its subevents make P true. Assuming that one cannot hold a p belief and a $\neg p$ belief at once, this is a contradiction.

(27) PFV [MAX [a think p
ass: $\exists e[\tau(e) \subseteq R \wedge dox(a, e) \subseteq p \wedge \forall e'[e \sqsubset e' \rightarrow \neg(dox(a, e') \subseteq p)]]$
psp: $\forall e'[dox(a, e') \subseteq p \vee dox(a, e') \subseteq \neg p]$
$\therefore \exists e[\tau(e) \subseteq R \wedge dox(a, e) \subseteq p \wedge \forall e'[e \sqsubset e' \rightarrow dox(a, e') \subseteq \neg p]]$

A different solution could say local accommodation of the EM presupposition—but then we’d have to say that it is systematically accommodated in the perfective. Why would that be?

Deriving appropriate NR disruption with scaleless implicatures In the previous section, we saw that the quantization conjunct is a source of disruption of NR in the perfective aspect. This will prove to be again the case if we take NR to be a scaleless implicature, but this time with a correct result, namely a plain meaning for negated perfective *think* with no NR.

In Jeretič (2022), following Jeretič (2021), NR is analyzed as a scaleless implicature, i.e. a strengthening from the reading given by the LF where *think* scopes below negation to the stronger NR reading. Scaleless implicatures are predicted by theories of scalar implicatures derived in the grammar, such as Bar-Lev and Fox (2020), whenever a quantifier has subdomain alternatives but no scalar alternative. In a nutshell, NR is the result of an exhaustivity operator EXH, as schematized below (aspectual operators ignored).

(28) ⟦ EXH [NEG [a think p]] ⟧ ≡ ⟦ a think NEG p ⟧

The existence of scalar alternatives to a quantifier are determined based on the available lexemes in a language. In English and French, the word for ‘think’ can qualify as lacking a scalar alternative, because it makes a universal claim about the doxastic set of an individual, and has no obvious existential scalemate in the lexicon. Subdomain alternatives, in contrast, are lexically specified. Typically, the definition of a subdomain alternative is an expression obtained by replacing the domain of a quantifier with a non-empty subset; however for *think*, the definition is more complicated, because its domain contains bound variables. For this, we follow Jeretič (2022) in arriving at subsets of the assignment-dependent domain via choice functions which pick out members of $\mathcal{P}(dox(x, e))/\varnothing$ (the set of non-empty subsets of $dox(x, e)$), for each possible assignment of x and e. Thus, the set of subdomain alternatives for *think* correspond to replacing $dox(x, e)$ with $F(\mathcal{P}(dox(x, e))/\varnothing)$ for all possible choice functions F.

(29) Alt(think) = $\{\lambda p.\lambda x \in D_e.\lambda e \in E_v.F(\mathcal{P}(dox(x, e))/\varnothing) \subseteq p \,|\, F$ is a choice function defined on $\mathcal{P}(dox(x, e))/\varnothing$ for all $e \in E_v$ and $x \in D_e\}$ (Jeretič, 2022)

[10] Perhaps we were wrong about our non-trivial assumption that EM affects the MAX conjunct. Indeed, we could have the presupposition quantify over the domain of events D shared with that of the main assertion, but not D', that of the quantized conjunct (the assertion remains consistent if we can have $D \subset D'$). However doing so derives the unattested meaning that either all belief states of a in R are $\neg p$, or there is a non-quantized p-belief state in R, shown here: $\forall e \in D[\tau(e) \subseteq R \wedge \forall e' \in D'[e \sqsubset e' \rightarrow \neg(dox(a, e') \subseteq p)] \rightarrow dox(a, e) \subseteq \neg p]$.

Following Bar-Lev and Fox (2020), implicatures are derived by applying an operator EXH to a sentence S and its alternative set $\text{Alt}(S)$. EXH asserts S, negates all Innocently Excludable (IE) alternatives (whose negation can be conjoined non-arbitrarily and consistently to S) and asserts all Innocently Includable (II) ones (which can be conjoined non-arbitrarily and consistently to IE-strengthened S). See formal definition in the Appendix.

We first show how NR is derived when EXH is applied to negated imperfective *think*. We begin with the LF in (30) before exhaustification, with its translation in (30a) and its set of alternatives in (30b). (31) is the result of applying EXH to (30) (see Appendix for full derivation).

(30) NEG [IPFV [a think p

a. **ass:** $\neg\exists e[dox(a,e) \subseteq p \wedge R \subseteq \tau(e)]$

b. **alts:** $\{\neg\exists e[F(\mathcal{P}(dox(a,e))/\varnothing) \subseteq p \wedge R \subseteq \tau(e)] \mid F$ is a choice function defined on $\mathcal{P}(dox(x,e))/\varnothing$ for all $e \in E$ and $x \in D\}$

(31) $[\![$ EXH [(30)][Alt((30))] $]\!] \equiv \neg\exists e[dox(a,e) \subseteq p \wedge R \subseteq \tau(e)] \wedge \boxed{\forall e[R \subseteq \tau(e) \rightarrow dox(a,e) \subseteq \neg p]}$

The derived inference states that all belief states of a running through R are $\neg p$-beliefs, which entails the NR inference. (Note we have the same result with the EM presupposition in (19).)

We now show the failure to derive NR with the perfective. (32) is the LF of a perfective *think* utterance before EXH applies. In (32a) is its translation, and in (32b) its alternatives.

(32) NEG [PFV [MAX [a think p

a. **ass:** $\neg\exists e[\tau(e) \subseteq R \wedge dox(a,e) \subseteq p \wedge \forall e'[e \sqsubset e' \rightarrow \neg(dox(a,e') \subseteq p)]]$
$\equiv \forall e[\tau(e) \subseteq R \wedge \forall e'[e \sqsubset e' \rightarrow \neg(dox(a,e') \subseteq p)] \rightarrow \neg(dox(a,e) \subseteq p)]$

b. **alts:** $\{\neg\exists e[F(\mathcal{P}(dox(a,e))) \subseteq p \wedge \tau(e) \subseteq R \wedge \forall e'[e \sqsubset e' \rightarrow \neg F(\mathcal{P}(dox(a,e'))) \subseteq p)]] \mid F$ is a choice function defined on $\mathcal{P}(dox(a,e))/\varnothing$ for all $e \in E\}$

The crucial detail in (32b) is that a subdomain alternative is defined by replacing *each* instance of *dox* with a given subset. This follows if we assume that the alternatives of an expression are obtained by pointwise function application from the alternatives of its parts. Thus, if MAX combines with P whose alternative set is Alt(P), the alternative set of MAX(P) will be obtained by slotting each alternative of P into each instance of the lambda-bound P in the expression for MAX. Recall that MAX, defined in (11b), selects for a predicate P and makes it appear twice in its assertion. This means that a given alternative of MAX(a think p) is obtained by having substituted an alternative of a think p into both of MAX's P-slots.

As also seen in the EM section, at these two places we have negated p-beliefs, which are thus candidates for NR. And indeed, EXH will locally derive NR at both places, as seen in (33).

(33) $[\![$EXH[(32)][Alt((32))]$]\!] \equiv \neg\exists e[dox(a,e) \subseteq p \wedge \tau(e) \subseteq R \wedge \forall e'[e \sqsubset e' \rightarrow \neg(dox(a,e') \subseteq p)]]$
$\wedge \boxed{\forall e[(\tau(e) \subseteq R \wedge \forall e'[e \sqsubset e' \rightarrow dox(a,e') \subseteq \neg p]) \rightarrow dox(a,e) \subseteq \neg p]}$

The complete resulting inference, instead of being NR, is a tautology, in the same way that the result with the EM presupposition in (26) is tautological (so we do not repeat the explanation here). However the difference between this result and the one obtained with the EM presupposition is that on the presuppositional view, the assertion itself together with the presupposition *is* a tautology, and the non-NR meaning is unrecoverable. Here on the other hand, we have the base negated perfective meaning left intact, and it is simply *conjoined* to the tautological inference (as one does with implicatures). We therefore are left with an expression equivalent to the unenriched meaning of a negated perfective thought report, as desired.

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Appendix

Definition of exh. We define our exhaustivity operator below, taken from Bar-Lev and Fox (2020).

(34) a. $IE(p, C) = \bigcap\{C' \subseteq C : C'$ is maximal & $\{\neg q : q \in C'\} \cup \{p\}$ is consistent$\}$
b. $II(p, C) = \bigcap\{C'' \subseteq C : C''$ is maximal & $\{r : r \in C''\} \cup \{p\} \cup \{\neg q : q \in IE(p, C)\}$ is consistent$\}$

(35) $[\![\textsc{exh}]\!](C)(p)(w) \equiv \forall q \in IE(p, C)[\neg q(w)] \wedge \forall r \in II(p, C)[r(w)]$

NR with negated imperfective thought reports.

(36) NEG [IPFV [a think p
a. **ass:** $\neg\exists e[R \subseteq \tau(e) \wedge dox(a, e) \subseteq p]$
$\equiv \forall e[R \subseteq \tau(e) \rightarrow \neg(dox(a, e) \subseteq p)]$
b. **psp:** $\forall e'[dox(a, e') \subseteq p \vee dox(a, e') \subseteq \neg p]$
c. $\therefore \forall e[R \subseteq \tau(e) \rightarrow dox(a, e) \subseteq \neg p]$

To show the derivation, we use an unrealistic model containing two eventualities associated with 2-world doxastic states: $dox(a, e_1) = \{w_1, w_2\}$, $dox(a, e_2) = \{w_3, w_4\}$. An example alternative of (36) evaluated with respect to this toy model is shown in (37).

(37) $\neg(\{w_1, w_2\} \subseteq p \wedge R \subseteq \tau(e_1) \vee \{w_3\} \subseteq p \wedge R \subseteq \tau(e_2))$

There are no IE alternatives: we can see this by attempting to exclude the strongest alternatives, i.e. those least likely to affect the result, which correspond to those obtained by replacing the domain with a singleton subset. The conjunction of the negation of these alternatives is equivalent to the negation of the prejacent. Excluding them would result in a contradiction, and excluding some but not all of these would be arbitrary; this means there are no IE alternatives.

(38) $[\{w_1\} \subseteq p \wedge R \subseteq \tau(e_1) \vee \{w_3\} \subseteq p \wedge R \subseteq \tau(e_2)]$
$\wedge[\{w_1\} \subseteq p \wedge R \subseteq \tau(e_1) \vee \{w_4\} \subseteq p \wedge R \subseteq \tau(e_2)]$
$\wedge[\{w_2\} \subseteq p \wedge R \subseteq \tau(e_1) \vee \{w_3\} \subseteq p \wedge R \subseteq \tau(e_2)]$
$\wedge[\{w_2\} \subseteq p \wedge R \subseteq \tau(e_1) \vee \{w_4\} \subseteq p \wedge R \subseteq \tau(e_2)]$
$\equiv \exists e[dox(a, e) \subseteq p \wedge R \subseteq \tau(e)]$

In contrast, all alternatives are II; we show in (39) the result of conjoining the same singleton-based alternatives to (36) (the non-singleton-based alternatives are also II; but their inclusion doesn't further affect the result, as we have already obtained a universal claim, the strongest result possible).

(39) $[\![$ EXH [(36)][Alt((36))] $]\!] \equiv \neg\exists e[dox(a,e) \subseteq p \wedge R \subseteq \tau(e)]$
$\wedge\neg(\{w_1\} \subseteq p \wedge R \subseteq \tau(e_1) \vee \{w_3\} \subseteq p \wedge R \subseteq \tau(e_2))$
$\wedge\neg(\{w_1\} \subseteq p \wedge R \subseteq \tau(e_1) \vee \{w_4\} \subseteq p \wedge R \subseteq \tau(e_2))$
$\wedge\neg(\{w_2\} \subseteq p \wedge R \subseteq \tau(e_1) \vee \{w_3\} \subseteq p \wedge R \subseteq \tau(e_2))$
$\wedge\neg(\{w_2\} \subseteq p \wedge R \subseteq \tau(e_1) \vee \{w_4\} \subseteq p \wedge R \subseteq \tau(e_2))$
$\equiv \neg\exists e[dox(a,e) \subseteq p \wedge R \subseteq \tau(e)] \wedge \forall e[R \subseteq \tau(e) \rightarrow dox(a,e) \subseteq \neg p]$

No NR with negated perfective thought reports.

(40) NEG [PFV [MAX [a think p
a. **ass:** $\neg\exists e[\tau(e) \subseteq R \wedge dox(a,e) \subseteq p \wedge \forall e'[e \sqsubset e' \rightarrow \neg(dox(a,e') \subseteq p)]$
$\equiv \forall e[\tau(e) \subseteq R \wedge \forall e'[e \sqsubset e' \rightarrow \neg(dox(a,e') \subseteq p)] \rightarrow \neg(dox(a,e) \subseteq p)]]$
b. **alts:** $\{\neg\exists e[F(\mathcal{P}(dox(a,e))) \subseteq p \wedge \tau(e) \subseteq R \wedge \forall e'[e \sqsubset e' \rightarrow \neg F(\mathcal{P}(dox(a,e'))) \subseteq p)]] \,|\, F$ is a choice function defined on $\mathcal{P}(dox(a,e))/\varnothing$ for all $e \in E\}$

Like above, no alternative is IE (left to the reader), but all alternatives are II. The effect of EXH is in (41); we show the inclusion of one alternative, the rest to be filled out.

(41) $[\![$EXH[(40)][Alt((40))]$]\!] \equiv \neg\exists e[dox(a,e) \subseteq p \wedge \tau(e) \subseteq R \wedge \forall e'[e \sqsubset e' \rightarrow \neg(dox(a,e') \subseteq p)]]$
$\wedge\neg(\{w_1\} \subseteq p \wedge \tau(e_1) \subseteq R \wedge \forall e'[e_1 \sqsubset e' \rightarrow \neg(\{w_1\} \subseteq p)]$
$\vee\{w_3\} \subseteq p \wedge \tau(e_2) \subseteq R \wedge \forall e'[e_2 \sqsubset e' \rightarrow \neg(\{w_3\} \subseteq p)])\wedge \ldots$
$\equiv \neg\exists e[dox(a,e) \subseteq p \wedge \tau(e) \subseteq R \wedge \forall e'[e \sqsubset e' \rightarrow \neg(dox(a,e') \subseteq p)]]$
$\wedge\forall e[\tau(e) \subseteq R \wedge \forall e'[e \sqsubset e' \rightarrow dox(a,e') \subseteq \neg p)] \rightarrow dox(a,e) \subseteq \neg p]$